Section 3.2: Subspaces of Vector Spaces

1. Let V be a vector space. A subset $W \subseteq V$ that is a vector space under the operations of V is called a **subspace** of V.
2. What examples of subspaces have we already seen?
3. **Theorem 3.10:** Let W be a subspace of a vector space V, and let $\mathbf{0}$ be the zero vector of V. Then $\mathbf{0} \in W$.
4. **Theorem 3.11:** (Subspace Test) Let W be a nonempty subset of a vector space V. Then W is a subspace of V if and only if W is closed under the operations of V.

Affine Subsets of Vector Spaces

1. Let W be a subspace of a vector space V, and let $\mathbf{x} \in V$. A set of the form
$$\mathbf{x} + W = \{\mathbf{x} + \mathbf{w} \; : \; \mathbf{w} \in W\}$$
is called an **affine subset** of V.
2. **Theorem 3.12:** Let A be an $m \times n$ matrix. The set of all solutions to the linear system $A\vec{\mathbf{x}} = \vec{0}$ is a subspace of $\mathbb{R}^n$.
3. **Theorem 3.18:** Let A be an $m \times n$ matrix and let $\vec{\mathbf{b}} \in \mathbb{R}^m$. The set of all solutions to the linear system $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$ is either the empty set or is an affine subset of $\mathbb{R}^n$.
(Note: There is a typo in the book in Theorem 3.18.)